Arithmetic and L-functions

November 12 to 16

Abstracts

Bruno ANGLÈS

L'arithmétique des t-modules d'Anderson

Cet exposé concerne les valeurs spéciales des séries L introduites par D. Goss en 1979. Les valeurs aux entiers positifs de certaines de ces séries sont étroitement liées aux objets introduits par G. Anderson en 1986 : les t-modules. En nous basant sur un travail en commun avec T. Ngo Dac et F. Tavares Ribeiro, nous présenterons des résultats récents sur ce sujet et nous clarifierons une Conjecture formulée par L. Taelman en 2009 .

Radan KUČERA
Masaryk University, Brno, Czech Republic

Annihilating ideal class groups via semispecial units

(joint work with Cornelius Greither and Hugo Chapdelaine)

This talk is devoted to a problem of finding annihilators of the ideal class group of the ring of integers of an abelian field, i.e., of a finite Galois extension of $\mathbb{Q}$ with an abelian Galois group. The oldest results in this direction go back to the end of 19th century and are based on Stickelberger's factorization of the principal ideal generated by a Gauss sum. This approach can give annihilators of the minus part of the class group of an imaginary abelian field.

In the late 1980's, F. Thaine found a new technique for constructing relations in ideal class groups. His method was almost immediately generalized by K. Rubin. Roughly speaking and considering only the easiest case of the base field being $\mathbb{Q}$, let K be a real abelian field with the Galois group $G = \mathrm{Gal}(K/\mathbb{Q})$ and let $\mathcal{O}_K^\times$ be the group of units of the ring $\mathcal{O}_K$ of integers of K, then (under suitable assumptions) any $\mathbb{Z}[G]$-linear map $\rho : \mathcal{O}_K^\times \to \mathbb{Z}[G]$ evaluated on a so-called special unit in K gives an annihilator of the ideal class group of $\mathcal{O}_K$. The usual source of special units is the Sinnott group of circular units : each Sinnott circular unit of K is special in K.

In a joint research with C. Greither we have slightly relaxed Rubin's definition of special units : we call our units to be semispecial. Each special unit is semispecial, but there are units which can be shown to be semispecial, even though we do not know whether they are special or not. For example, if the genus field of K in the narrow sense is real, then any unit of K, which is a circular unit of a larger abelian field containing K, is semispecial in K. Moreover, we have modified Rubin's machinery to accept semispecial units : (under suitable assumptions) any $\mathbb{Z}[G]$-linear map $\rho : \mathcal{O}_K^\times \to \mathbb{Z}[G]$ evaluated on a semispecial unit in K gives an annihilator of the ideal class group of $\mathcal{O}_K$.

In the talk, explicit infinite families of real abelian fields such that this approach gives an annihilation result, which is stronger than the result obtained by the standard application of Rubin's machinery, will be mentioned.
An analogical situation, where the base field is an imaginary quadratic field and where the role of circular units is played by elliptic units, was the subject of a joint research with H. Chapdelaine and will be touched on during the talk as well.

Jilali ASSIM

Galois codescent for the motivic tame kernels

(Joint work with A. Chazad Movahhedi)

Let L/F be a Galois extension of number fields with an arbitrary Galois group G. We give an explicit description of the kernel of the natural map on motivic tame kernels $H^2_{\mathcal{M}}(\mathcal{O}_L, \mathbf{Z}(i))_G \to H^2_{\mathcal{M}}(\mathcal{O}_F, \mathbf{Z}(i))$. Using the link between motivic cohomology and K-theory, we have a general genus formula for all even K-groups $K_{2i-2}(\mathcal{O}_F)$ of the ring of integers. As a by-product, we also obtain lower bounds for the order of the kernel and cokernel of the functorial map $H^2_{\mathcal{M}}(F, \mathbf{Z}(i)) \to H^2_{\mathcal{M}}(L, \mathbf{Z}(i))^G$.

Florent JOUVE

Chebyshev's bias in Galois groups of number fields

In a 1853 letter, Chebyshev observed that in most intervals $[2, x]$ there are more primes of the form $4n+3$ than of the form $4n+1$. Many generalizations of this bias phenomenon have been studied. In this talk we will discuss joint work with D. Fiorilli on Chebyshev's bias in the context of the Chebotarev density theorem. Our focus will be on particular families that either exhibit a surprising behavior as far as Chebyshev's bias is concerned or that are simple enough to enable a very precise computation of the group theoretic and ramification theoretic invariants that come into play in our analysis. Precisely the emphasis will be on some families of abelian, dihedral, or radical extensions of $\mathbb{Q}$ as well as families of Hilbert class fields H_d of quadratic fields K_d either seen as extensions of $\mathbb{Q}$ or of K_d.

Werner BLEY

The local epsilon constant conjecture for unramified twists of $\mathbb{Z}_p(1)$

Let N/K be a finite Galois extension of p-adic number fields. We will give an explicit reformulation of the equivariant local epsilon constant conjecture, formulated previously by various authors (Kato, Benois and Berger, Fukaya and Kato and others), in the special case of certain 1-dimensional unramified twists of $\mathbb{Z}_p(1)$. In joint work with A.Cobbe we have shown the validity of this conjecture for certain wildly and weakly ramified abelian extensions N/K. Comparing the twisted conjecture to a conjecture of Breuning we obtain an explicit conjectural description of a certain Euler characteristic related to local fundamental classes which already occurs in the formulation of Chinburg's Ω_2-conjecture. This part of the talk concerns joint work in progress with D.Burns.

Maria Rosaria PATI

Generalized Heegner cycles and derivatives of p-adic L-functions

Let f be a newform of even weight $k_0 > 2$ on $\Gamma_0(N)$. Let K be an imaginary quadratic field of discriminant prime to N such that the number of primes of N which are inert in K is even, and let p be one of these primes. We may attach to f and K a p-adic L-function of the weight variable k. Our result is a formula relating the first derivative of this p-adic L-function at $k = k_0$ to the Abel-Jacobi image of a so-called generalized Heegner cycle.

Denis BENOIS

TBA

Özlem IMAMOGLU

Integrals of modular functions, Markov geodesics and a conjecture of Kaneko

In this talk I will report on joint work with P. Bengoechea in which we study the "values" of modular functions at Markov numbers. The values of modular functions at real quadratic irrationalities, which can be thought as the analog of values of modular functions at CM points, are defined in terms of cycle integrals along closed geodesics. I will start the talk with an introduction to cycle integrals, Markov numbers and their properties and then give the conjectures of Kaneko on the values of modular functions along closed geodesics in general and along the Markov tree in particular. I will finish with the proof of some of these conjectures.

Marc HINDRY

Analogues of Brauer-Siegel theorem in arithmetic geometry

The Brauer-Siegel theorem says that the product of the class number by the units regulator of a number behaves like the square root of its discriminant, under mild conditions (example given : when the degree of the field over Q remains bounded). We discuss ample generalisations where at least a similar upper bound can be proven for the product of the cardinality of a local-global obstruction group by a regulator. We will also discuss lower bounds and, though we are unable to prove anything, will suggest that the behaviour is different according to the location of the relevant special value of the associated zeta function (center or edge of the critical strip)

Douglas ULMER

The Brauer-Siegel ratio and the dimension of the Tate-Shafarevich group

In the context of abelian varieties over global function fields, I will explain what the Birch and Swinnerton-Dyer conjecture leads us to expect for the behavior of the Tate-Shafarevich group as a function of the constant field. In some cases, this expectation can be verified, and this leads to calculations of Hindry's Brauer-Siegel ratio in many interesting cases.

Richard GRIFFON

Elliptic curves with large Tate—Shafarevich groups and Siegel's theorem

A classical theorem of Siegel's provides estimate on the class-number of imaginary quadratic number fields in terms of their discriminant. We consider the analogous setting of rank 0 elliptic curves over $\mathbb{F}_q(t)$: if E is such a curve, one is interested in finding upper and lower bounds on the order of the Tate—Shafarevich group of E (assuming it is finite) in terms of simpler invariants of E, e.g. its height or the degree of its conductor. In general, this is an open problem. In this talk, I will explain the analogy between the two settings and report on recent work (joint with Guus de Wit) where we studied a certain Artin—Schreier sequence of rank 0 elliptic curves for which we found good unconditional bounds on the order of Sha. In particular, this provides examples of elliptic curves with large Tate—Shafarevich groups over $\mathbb{F}_q(t)$. Our bounds show that the analogue of Siegel's theorem holds for this family of curves.

Dimitar DETCHEV

Special cycles on Shimura varieties for U(n, 1) x U(n-1, 1) and the Bloch–Kato conjecture for conjugate-dual Galois representations

Using a recently constructed Euler system of special cycles on unitary groups, we show how to obtain new results towards the Bloch–Kato conjecture for conjugate-dual Galois representations appearing in the middle-degree cohomology of the ambient unitary Shimura variety. In addition, we explain how to use recent vertical norm-compatibility result due to Boumasmoud, Brooks and Jetchev in order to obtain novel anticyclotomic Iwasawa-theoretic results.

Anna CADORET

Weil II ultraproduit pour les courbes et $\mathbb{Z}_\ell$-compagnons.

On définira une catégorie de faisceaux lisses à coefficients ultraproduits et on construira dans ce cadre un formalisme partiel des poids de Frobenii parallèle à celui de Weil II ; on démontrera notamment le théorème fondamental de Weil II pour les courbes. En combinant ce résultat à des arguments de nature géométrique (Bertini, pinceaux de Lefschetz etc.) on en déduira (sans restriction sur la dimension de la variété) la plupart des corollaires classiques de Weil 2 (pureté, semisimplicité géométrique, Cebotarev faible). On donnera des applications aux modèles entiers dans les systèmes compatibles de faisceaux l-adiques lisses : unicité asympotique des modèles entiers, semisimplicité géométrique asympotique de la réduction modulo-l, généralisation du théorème de Gabber sur la torsion des images directes supérieures etc. Ces résultats permettent également de démontrer la correspondance de Langlands à coefficients ultraproduits et une forme asymptotique de la correspondance de Langlands modulo-l, impliquant l'existence et l'unicité asympotique du relèvement dans la conjecture de de Jong.

Wei HO

Integral points on elliptic curves

Ignazio LONGHI

Closed subsets of the finite adeles

Questions related to the size of some subset of $\mathbb{Z}$ (for example, how many prime numbers ; or the probability that two integers are coprime) are traditionally studied by means of analytic number theory. However, one can try and approach these kind of questions in a more algebraic way, by looking at the profinite completion of this subset (that is, its closure in the finite adeles) and computing its Haar measure. I will describe some work in progress (partially done with my students Jiali Du, Yunzhu Mu and Francesco Saettone ; and more recently with Luca Demangos) on this circle of ideas, discussing a few results and many questions which arise from this point of view.

Chazad Movahhedi

p-rational number fields : old and new results

Let F be a number field and p a rational prime number. Denote by F_{S_p} the maximal p-extension of F which is unramified outside p-adic primes. The field F is called p-rational when the Galois group $Gal(F_{S_p}/F)$ is a free pro-p-group. This notion was introduced and studied in my 1988 Ph.D. thesis. My lecture concerns these p-rational fields, which have been recently revisited by Greenberg in his study of some representations of the absolute Galois group of the field of rational numbers.

Michael TSFASMAN

Dense sphere packings : state of the art and algebraic geometry constructions

(CNRS - LMV & IITP RAS & Independent University of Moscow)
mtsfasman@yandex.ru

How dense can we pack equal spheres in the Euclidean space $\mathbb{R}^N$? The question looks natural and is treated by humanity at least since the end of

16th century. The first four hundred years of research gave us the answers only in dimensions 1, 2, and 3. Quite recently, the answers for $N = 8$ and $N = 24$ — that we always presumed to be true — were proved by an elegant technique using modular forms [1], [2].
If we restrict ourselves to the easier situation when the centers of the spheres form a lattice (an additive subgroup of $\mathbb{R}^N$) the answer is known for N from 1 to 8, and, of course, for $N = 24$. Not too much either ...
We have to ask easier questions. Can we bound the density and how ? Which constructions give us packings that, if not being the best, are however dense enough ?
Number fields and curves over finite fields provide lovely constructions [3]. To find out their densities we need to know a lot about our algebraic geometry objects. In particular, we study their zeta-functions.
As usual, when we do not know the answer for a given N we try to look at what happens when $N \to \infty$. This time we need to understand the asymptotic behaviour of zeta-functions when the genus tends to ∞, cf. [4], [5], [6], [7].
My dream is a nice theory of limit objects such as projective limits of curves or infinite extensions of $\mathbb{Q}$, as yet we are very far from it.
Another great challenge is to construct lattice sphere packings that are denser that those given by a random construction (so-called Minkowski bound).

Références

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